JOURNAL OF COMPUTATIONAL PHYSICS 60, 558–560 (1985)

Note

The Reliability of Calculations with the RSF Algorithm*†

In the preceeding paper [1], Eastwood and Hopcraft investigate the numerical stability properties of the intial-value scheme used in our RSF code [2]. They present approximate von Neumann stability analyses and comment on the implications of this for the efficiency and correctness of RSF results. Their analysis relates numerical instabilities to physical conditions such as the magnitude and orientation of flow velocity. Such approximate numerical stability analyses do not, however, provide conclusive and general stability rules particularly for a nonlinear scheme. The numerical stability analysis is based on the linear part of the numerical scheme. For the usual physics applications the time evolution is dominated by the nonlinear terms. Consequently, our approach is to take a more empirical point of view. An adaptive timestep control routine analyzes certain amplitudes for very short wavelength components. When these are observed to grow, the timestep is automatically reduced. This, in practice, avoids all but the fastest growing numerical instabilities. This approach has the additional virtue of arriving at a timestep which is near the maximum feasible value, thus providing the greatest efficiency for this scheme.

In spite of the success of this adaptive timestep algorithm, it is essential to verify the numerical stability of results obtained. We primarily use two techniques to accomplish this. We compare the results of up to four different numerical schemes. Numerically unstable results from different schemes would differ, so this provides a test for stability. For example, RSF results have been compared to the results obtained with a scheme [3] in which all of the linear terms are treated implicity and which therefore has no stability restriction on the time step in the linear regime. The other type of test is to rerun a calculation with the same scheme but with a different timestep. Since the growth of a numerical instability depends on the timestep, results which are independent of the timestep do not contain significant numerically unstable components. Such convergence studies have shown that the adaptive timestep scheme is very successful at avoiding numerical instabilities. These tests have been applied systematically in all the physics applications we have made, thus excluding any contamination of results due to numerical instabilities. When the timestep is not adaptively reduced, a numerical instability having an enormous growth rate appears.

* Research sponsored by the Office of Fusion under Contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

0021-9991/85 $3.00

The results of an extensive program of numerical tests of the RSF algorithm were reported in [4]. Both 2D and 3D nonlinear tearing mode calculations were considered. [4] addresses some questions raised earlier by Eastwood and Hopcraft [5], but goes on to consider each potential source of numerical error: numerical stability, roundoff, and several sources of truncation error. The conclusions are that the potential sources of numerical error in RSF are well understood and that the published RSF results are numerically stable and converged. In addition, we considered [3] the relative computational efficiency, under a variety of physical situations, of RSF and a scheme in which all of the linear terms are implicit.

While we agree with many of the conclusions of [1], we also have some significantly different interpretations. The claim that physical processes may become confused with numerical effects is only true if care is not taken to verify the numerical stability by appropriate methods. Since we and other researchers in this field have routinely done this, this argument does not apply. Figure 2a of [1] illustrates that numerically contaminated solutions can be obtained, but as Fig. 2c shows, such a problem can be easily detected. Use of the overall energy conservation as a check of validity is proposed in [1] and, more strongly, in [5]. We have shown [4] that this is a dangerously misleading diagnostic unless interpreted with great care. In [2] we indicated an appropriate initial timestep size. This is Eq. (16) of [1]. We have never claimed that this timestep is a sufficient condition, which is why we have employed the adaptive timestep control described above. In the $S \to \infty$ limit, the RSF equations become singular and thus the stability of the scheme in this limit is a moot point.

The strict requirement that the amplification factor λ not lie outside the unit circle is overly conservative, especially when the equations are not elliptical and when they admit growing physical solutions. A magnitude of λ slightly greater than unity corresponds to a slowly growing numerical instability. It is only necessary that its amplitude remain small with respect to the physical solution (as is accomplished by the adaptive timestep algorithm). In any case it is pointless to develop very sophisticated numerical stability criteria for the linear equation, because the real limitation on the timestep arises from the nonlinear terms.

We concur with [1] on numerous points, including the desirability of a better understanding of these numerical schemes with the goal of improved domains of numerical stability. We agree that schemes exist which in certain situations are computationally more efficient than RSF. In fact, we have developed and used such schemes [3, 4] since the time that the RSF algorithm was first developed in 1978.

References

1. J. W. Eastwood and K. I. Hopcraft, *J. Comput. Phys.* **60** (1985), 549.
2. H. R. Hicks, B. A. Carreras, J. A. Holmes, D. K. Lee, and B. V. Waddell, *J. Comput. Phys.* **44**, (1981), 46; *J. Comput. Phys.* **53** (1984), 205.

3. L. GARCIA, H. R. HICKS, B. A. CARRERAS, AND J. A. HOLMES, "3-D Nonlinear MHD Calculations with Drift and Thermal Force Effects," ORNL/TM-9499, Oak Ridge National Labs., Oak Ridge, Tenn.
4. H. R. HICKS, B. A. CARRERAS, L. GARCIA, J. A. HOLMES, "The Reliability of Initial-Value MHD Calculations of Tokamak Disruptions," ORNL/TM-9127, Oak Ridge National Labs., Oak Ridge, Tenn., June 1984.
5. J. W. EASTWOOD AND K. I. HOPCRAFT, *Bull. Amer. Phys. Soc.* **28** (1983), 1207.

RECEIVED February 5, 1985; accepted April 8, 1985

H. R. HICKS, B. A. CARRERAS, AND J. A. HOLMES
Oak Ridge National Laboratory,
Oak Ridge, Tennessee 37831